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THE HEAT TRANSFER COEFFICIENT FOR FLOW IN A PIPE

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Abstract—The paper records new data on the local heat transfer coefficient in a straight pipe remote from the entrance. Water and air were used, at Reynolds numbers from 300–100 000; the pipes were horizontal, of seven different diameters from 1·27 to 5·08 cm. Temperature differences were small.

Results for laminar and turbulent flow are compared with conventional formulae. For laminar flow they are considerably higher than predicted by elementary theory because of natural convection effects; these are considered in some detail.

A PROGRAMME of experimental work on heat transfer between a pipe and fluid flowing through it has been in progress at NEL for some years. The object has been to study in detail the variation in the local heat transfer coefficient arising from disturbances to the fluid flow caused by an abrupt change in the diameter of the pipe, by a bend or by an abrupt elbow. The fluids used have been air and water. The work has been somewhat similar to that described by Petukhov and Krasnoschekov [1].

As a by-product of these experiments a considerable amount of data has been accumulated on the value of the local heat transfer coefficient at positions remote from such disturbances, where the variation along the pipe has fallen to a negligible level. Data of this type may be compared with the many formulae which are available for calculating the local coefficient under such circumstances, or the average coefficient for a long pipe, and it is felt that, as there does not appear as yet to be any final agreement on the best formulae to be used, the data may be worth putting on record. At the lower Reynolds numbers interesting observations on the effect of natural convection have been made.

The apparatus used in these experiments was in all important characteristics similar to that described in an earlier paper [2]. The various experimental pipes were heated by the passage of electricity along the pipes themselves; pipe temperatures were obtained by using thermocouples attached to the outside at a series of positions along the pipe, the corresponding temperatures at the inside surface being deduced by calculation. A number of thermocouples were attached to the pipe at each position in order to investigate the distribution of temperature around the periphery. For most purposes, the heat transfer coefficients calculated for each position have been based on the average of the readings of the thermocouples at that position. Local bulk fluid temperatures were calculated on the basis of measurements of inlet temperature, rate of flow of fluid, and heat input up to the point in question; outlet temperatures were also measured, and were used as a check on accuracy. Heat fluxes and pipe-to-fluid temperature differences were kept as low as possible, consistent with reasonable accuracy of measurement, in order to avoid complications due to variation of physical properties with temperature. Precautions were taken to avoid errors due to fouling and, when water was used, deposition of bubbles of air. Experiment and calculation were used to make the necessary corrections for the flow of heat from the pipe to the outside atmosphere, and along the heavy copper leads conveying the electric current. Results were obtained with pipes of the following internal diameters: 0·5, 0·6, 0·75, 0·8, 1·0, 1·5 and 2·0 inches (1·27–5·08 cm). The range of Reynolds number explored extended from 100 000 down to 300; for values of *Re* less than about 5000, however, precision became increasingly difficult to obtain, first because of instability of flow in the neighbourhood of the critical Reynolds number, and, for still lower values of *Re*, because of effects due to natural convection, including the development of substantial differences in temperature between the top and bottom of the pipe. With air at low Reynolds numbers, the corrections were of comparable magnitude to the true heat transfer, and consequently no worthwhile results were obtained with laminar flow.

Each observation was reduced to a Nusselt number associated with a Reynolds number and Prandtl number (*Nu*, *Re* and *Pr* respectively), the physical properties having been taken at the bulk temperature. The range of variation of *Pr* in the experiments with air was negligible and a common value of 0·7 may be taken. For the tests with water, values from 4 to 12 were involved, the majority being around 8. Many proposals have been made for empirically representing the effect of the variation of Prandtl number; most involve the use of Pr^n, where n is given various values; for turbulent flow, they are generally in the neighbourhood of 0·4. The procedure has been adopted here of presenting the results in the form of a graph of $Nu/Pr^{0.4}$ against *Re*. Whether or not the choice of $Pr^{0.4}$ is the best possible, it should be sufficient to eliminate the comparatively small effect due to Prandtl number variations in the tests with water. The results are presented in this way in Fig. 1.

We consider first the data obtained with turbulent flow, for Reynolds numbers of about 8000 upwards. It will be seen that the results with water are closely correlated but that those obtained with air are not reconciled with those for water. As already mentioned, there are a variety of formulae which can be compared with

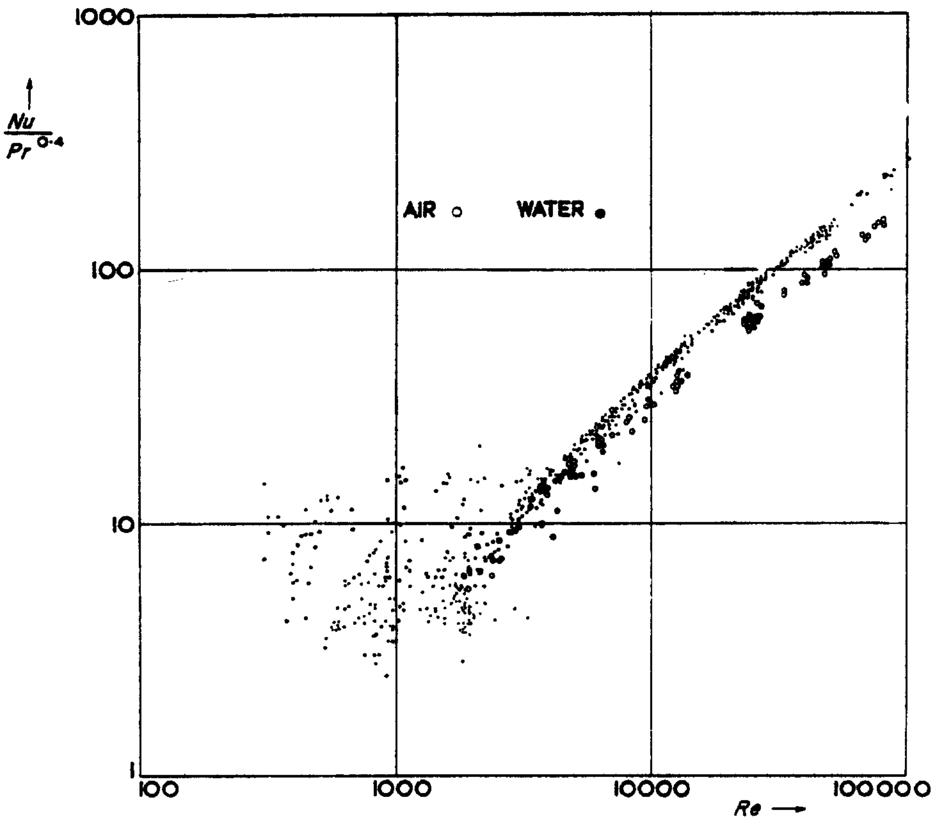

FIG. 1. Correlated experimental results, for air and water, obtained with pipes of diameters varying from 0·5 to 2·0 in. (1·27–5·08 cm).

these results. This is not the place for an exhaustive analysis, but two important examples will be briefly considered. First, there is the familiar empirical equation [3, p. 145]:

$$Nu = 0{\cdot}023 Re^{0{\cdot}8} \times Pr^{0{\cdot}4},$$

the physical constants being taken at the bulk temperature. A line corresponding to this equation is drawn in Fig. 2, which presents the results for turbulent flow on a larger scale. This equation can evidently serve as a compromise between the two sets of data for water and air, but does not represent either precisely. The slope of the line is also not perfectly in accordance with the results, but within the range of Reynolds numbers investigated the discrepancy is not serious. This type of formula has from its simplicity much to recommend it but, from the present evidence, different values of the coefficient would be required for air and for water, in the neighbourhood of 0·018 and 0·026 respectively. Other workers have proposed 0·021 and 0·027 [4, p. 219]. An improvement could be effected by changing the index of the Prandtl number, but a very substantial change would be needed.

The other formula which will be considered is of a completely different type, more satisfactory in derivation but less convenient in use; it is that due to Martinelli, based on refinements of Reynolds' analogy (see McAdams [4, p. 212]):

$$\frac{Nu}{Re\,.\,Pr} = \frac{\sqrt{(f/2)}}{5\varphi\left\{Pr + \ln(1+5Pr) + 0{\cdot}5Pr\,.\,\ln\left[\frac{Re}{60}\sqrt{\left(\frac{f}{2}\right)}\right]\right\}}$$

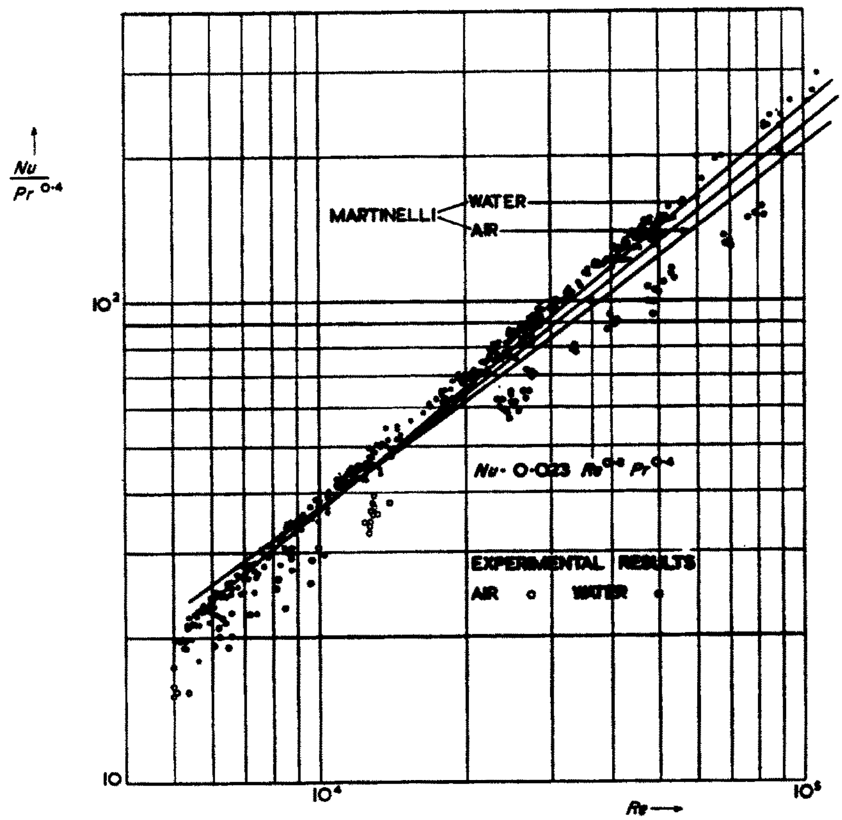

FIG. 2. Results for turbulent flow.

where f is the friction factor, and φ is a temperature-difference ratio. On the present method of plotting, this equation affords a variation with Prandtl number, and in Fig. 2 lines are drawn representing Prandtl numbers of 0·7 and 8·0, i.e. air and an average value for water. The line for water agrees quite well with the experimental data, both in slope and position, over the whole range of Reynolds number. The line for air diverges from that for water in the right direction, and at the higher Reynolds numbers has the right slope. Throughout the range, however, it predicts Nusselt numbers which are a good deal higher than the experimental values, and at the lower end it passes above the line for water, in contradiction to the experimental observations, which still remain below. Similar results are predicted by the various formulae of similar type developed by Kutateladze [3, p. 143].

The results obtained in the transitional region do not lend themselves to detailed analysis because of the instability already referred to. This produced fluctuating temperatures, the extent of the fluctuations being small at the higher Reynolds numbers: fluctuations were not detected above about $Re = 9000$. At lower Reynolds numbers the extent of the fluctuations increased and reached a magnitude of about ± 10 per cent in the region of $Re = 3000$. Much more stable results were obtained at such Reynolds numbers if the flow was deliberately "tripped" into turbulence by means of an artificial obstruction introduced near the entrance to the pipe. It will be observed from Fig. 1 that the local Nusselt number varies very sharply with Reynolds number in this range, in a similar manner to the well-known variation of the average Nusselt number. (See, for example, Fishenden and Saunders [5, p. 110].)

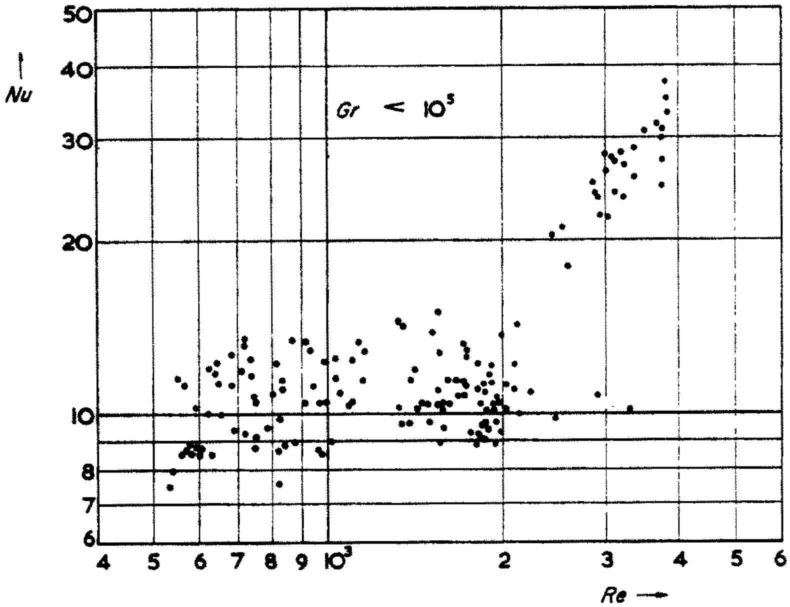

FIG. 3. Results for low Reynolds numbers with the Grashof number less than 10^5.

For Reynolds numbers below about 2300 the results were quite stable. As will be seen from Fig. 1, however, the Nusselt numbers obtained varied very widely; two factors which were clearly involved in this variation were the diameter of the pipe and the rate of heat flux, and it is obvious that natural convection was responsible. The scatter is reduced if limits are placed on the magnitude of the Grashof number; Fig. 3 shows the variation of Nu with Re

for those tests in which the Grashof number was less than 10^5. It appears that there is no consistent variation with Reynolds number until a departure from laminar flow occurs at about $Re = 2300$; this in accordance with theory which indicates (in the absence of natural convection) a constant value of 4·36 for the Nusselt number. The values measured are, however, in the region of 10, which is much greater than the theoretical figure, and indicates that even at the lowest range of Grashof number encountered the effect of natural convection is very pronounced.

The predominance of Grashof number effects and the probable lack of variation with Reynolds number suggest that better correlation of the laminar flow results would be provided by plotting Nu against Gr. This has been done in Fig. 4, which shows that this method of correlation is reasonably successful. The results may

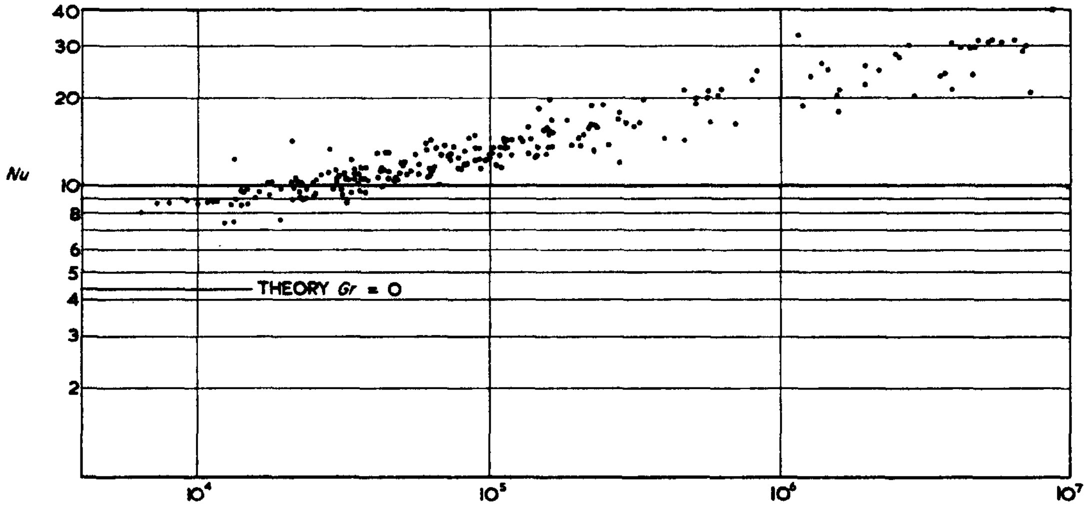

FIG. 4. Results for laminar flow correlated against Grashof number.

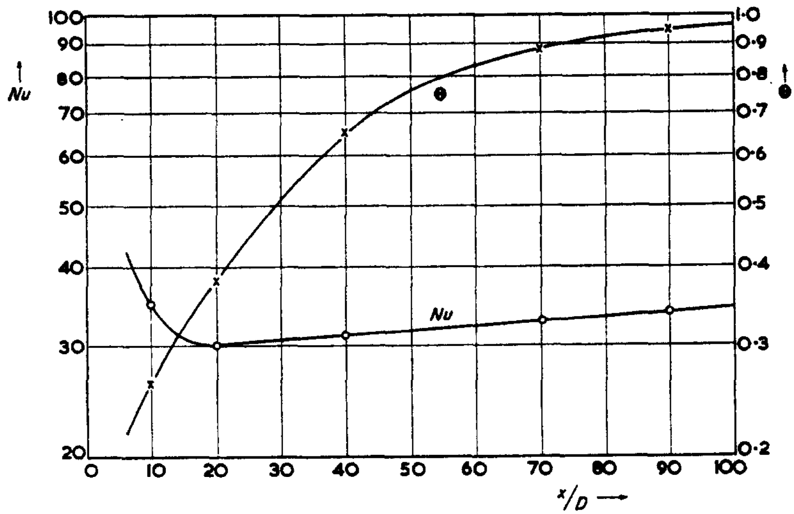

FIG. 5. Variation of Nusselt number and top/bottom temperature difference along a pipe, at a Reynolds number of 2100.

be approximately represented by the equation $Nu = 4{\cdot}36\ (1 + 0{\cdot}06\ Gr^{0{\cdot}3})$.

Another feature of some interest which arises at low Reynolds numbers is the development of a substantial difference in temperature between the top and bottom of the experimental pipe, indicating a gradual stratification of the warmer water above the cooler water in the pipe. In Fig. 5 the Nusselt number and the top-to-bottom temperature difference, θ, (in dimensionless form), are plotted against position along the pipe (measured in diameters), showing how the stratification effect develops and revealing that it has little influence on the value of the Nusselt number. The magnitude of the effect is governed by the dimensions of the pipe and the thermal conductivity of the material of which it is made, since the flow of heat by conduction in the pipe wall tends to reduce it. If the same temperature difference, θ, is plotted against the Grashof number, the results fall into compact groups according to the particular pipes with which they were obtained. For a given limited range of Grashof number a rough correlation can be made between θ and a figure indicating the amount of heat which would flow from top to bottom of unit length of the pipe under unit temperature difference.

The writer is not aware of any theoretical or experimental work which can be directly compared with the present data for laminar flow. Qualitatively, of course, the results have been familiar for many years. Morton [6], deals specifically with this problem, but his work is concerned with very small Grashof numbers.

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Résumé—L'article rassemble des données nouvelles sur le coefficient de transmission de chaleur dans un tuyau droit loin de l'entrée. L'étude a été faite avec de l'air et de l'eau, des nombres de Reynolds de 300 à 100.000, des tuyaux horizontaux de diàmètres échelonnés entre 1,27 et 5,08 cm. Les différences de températures étaient petites.

Les résultats des écoulements laminaire et turbulent sont comparés à ceux que donnent les formules habituelles. Pour un écoulement laminaire, ils sont considérablement plus élevés que ceux prévus par la théorie à cause des effets de convection libre, ceci est étudié en détail.

Zusammenfassung—Für den örtlichen Wärmeübergangskoeffizienten in einem geraden Rohr, in einiger Entfernung vom Einlauf werden neue Daten gebracht. Als Versuchsmedium diente Wasser und Luft bei Reynoldszahlen von 300–100 000 in waagrechten Rohren mit sieben verschiedenen Durchmessern von 1,27 bis 5,08 cm. Die Temperaturdifferenzen waren klein.

Die Ergebnisse für laminare und turbulente Strömung wurden mit bekannten Formeln verglichen. Für laminare Strömung liegen diese Ergebnisse beträchtlich über den Werten der Elementartheorie. Dies rührt vom Einfluss der natürlichen Konvektion her und wurde ebenfalls untersucht.

Аннотация—В статье излагаются результаты экспериментальных исследований по определению коэффициента теплообмена при течении в трубах воды и воздуха в диапазоне изменения числа Рейнольдса от 300 до 100 000.

Проведенные исследования показали, что при $Re > 8000$ опытные данные для воды и воздуха не согласуются между собой. При $Re < 8000$ опытные данные для воды и воздуха могут быть описаны одной приближенной формулой.